Surface Reconstruction from Point Clouds

Myungjoo Kang[1]
[1]Seoul National University
e-mail : mkang@snu.ac.kr

1 Introduction

We introduce a sophisticated deep learning approach to reconstruct surfaces from unorganized point clouds. Utilizing an implicit surface representation as a level set function, our method guarantees watertight results and adapts seamlessly to various topologies. We employ the p-Poisson equation to accurately learn the signed distance function (SDF), enhancing precision through a variable splitting strategy that incorporates the SDF gradient as an auxiliary variable. Furthermore, we enforce a curl-free condition on the auxiliary variable to leverage the irrotational nature of conservative vector fields. Our numerical results demonstrate that this strategic use of partial differential equations and key vector field characteristics efficiently reconstructs high-quality surfaces without the need for prior surface knowledge. We propose a new framework for surface reconstruction based on solving the p-Poisson equation using physics-informed neural networks (PINNs)[2].

2 Motivation

While traditional surface reconstruction approaches exist[1], such as Poisson surface reconstruction and level set methods, they often require additional information, such as normal vectors, or impose strong assumptions on the surface smoothness. More recently, implicit neural representations (INRs) have emerged as a promising direction. These models use neural networks to represent the signed distance function (SDF) of a surface and reconstruct it by taking the zero-level-set. However, most SDF-based methods enforce the Eikonal equation, whose solution is not unique. To guide convergence, these methods often require ground-truth normal supervision, which is not always available and can degrade performance when noisy. Furthermore, the use of second-order derivatives in the loss function for physics-informed training introduces optimization difficulties.

3 Limitations of Eikonal-based SDF Learning

Conventional INRs approximate the SDF by enforcing the Eikonal equation $\|\nabla u\| = 1$. However, the Eikonal equation admits multiple solutions, leading to ambiguity in training. This issue is often mitigated using additional supervision, such as ground-truth surface normals, but this introduces reliance on high-quality annotations and may compromise the reconstruction quality when annotations are inaccurate.

4 p-Poisson Equation Formulation

To address the non-uniqueness issue, we adopt the p-Poisson equation[3]:

$$-\nabla \cdot (|\nabla u|^{p-2} \nabla u) = 1$$

Proceedings of the 2025 Annual Meeting of the Japan Society for Industrial and Applied Mathematics

As $p \to \infty$, the unique solution u to this equation converges to the SDF[4]. This formulation is advantageous as it provides a principled way to enforce geometric consistency while ensuring a unique solution. It also opens the door to optimization strategies that avoid high-order derivatives.

5 Variable Splitting Strategy and Curl-Free Regularization

To simplify the training and reduce the computational burden, we adopt a variable splitting method by introducing an auxiliary variable $G = \nabla u$. This allows us to reformulate the PINN loss using only first-order derivatives, leading to:

$$\mathcal{L}(u, G) = \|G - \nabla u\|^2 + \left\|\nabla \cdot (|G|^{p-2}G) + 1\right\|^2$$

This strategy allows the model to learn the PDE constraint more effectively, avoids vanishing gradients from second-order derivatives, and facilitates training with large values of p.
To ensure G remains a conservative vector field (i.e., gradient of a scalar function), we introduce a soft constraint enforcing $\nabla \times G = 0$. This curl-free condition enhances the alignment between G and ∇u, improving the model' s convergence and geometric accuracy.

6 Advantages of Our Framework

- **Normal-Free Learning:** Unlike traditional methods, our model does not require surface normals, reducing dependency on external data.
- **Efficient Optimization:** By avoiding second-order derivatives, the training is stable and efficient even with large p.
- **Mathematical Rigor:** The p-Poisson formulation ensures uniqueness and convergence of the learned SDF.
- **Extension to UDFs:** Our method naturally extends to open surface reconstruction by learning unsigned distance functions.

References

[1] H. K. Zhao, S. Osher, B. Merriman, B, and M. Kang. Implicit and nonparametric shape reconstruction from unorganized data using a variational level set method. Comput. Vis. Image Und. 80(3), 295–314, Elsevier, 2000.

[2] M. Raissi, P. Perdikaris, G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, Journal of Computational Physics, 2019.

[3] Y. Park, T. Lee, J. Hahn, and M. Kang, p-Poisson surface reconstruction in curl-free flow from point clouds, Advances in Neural Information Processing Systems, 2023.

[4] Y. Park, C. Song, J. Hahn, and M. Kang, ReSDF: Redistancing implicit surfaces using neural networks, journal of Computational Physics, 2024.

Application of high-dimensional statistics to single-cell biology

Yusuke Imoto[1]
[1]Kyoto University
e-mail : imoto.yusuke.4e@kyoto-u.ac.jp

1 Introduction

Single-cell RNA sequencing (scRNA-seq) obtains gene-level expression per cell, yielding high-dimensional (≈ 20,000 dimensions) data. High-dimensional statistics studies the mathematical behavior of estimators when dimension is large, revealing phenomena such as sample–population mismatches (**the curse of dimensionality**) that do not occur in low-dimensional settings [1]. Consequently, conventional scRNA-seq analyses may overlook true biological structures. In this study, we develop **RECODE** (resolution of the curse of dimensionality), a noise-reduction method for scRNA-seq data based on eigenvalue correction for high-dimensional data proposed by Yata and Aoshima [2, 3]. In this talk, we present an overview of RECODE and discuss recent extensions [4, 5, 6].

2 High-dimensional statistics and the curse of dimensionality

When observational noise is present in high-dimensional data, quantities computed from the data, such as distances, fail to match their true values due to the accumulation of noise; this phenomenon is known as the curse of dimensionality [1]. Because it also affects correlations, eigenvalues, and other key statistics, the curse of dimensionality poses a fundamental challenge for almost all analysis tools for high-dimensional data.

To overcome these issues, the field of high-dimensional statistics has been developed. Classical asymptotic statistics assumes fixed dimension and increasing sample size, whereas high-dimensional statistics studies the joint asymptotic behavior of both the dimension d and the sample size n as they grow large.

$$\begin{aligned}
&\textbf{Asymptotic statistics:} && s_{n,d} \xrightarrow{p} s^{\mathrm{true}}, \quad n \to \infty,\ d: \text{fixed}.\\
&\textbf{High-dimensional statistics:} && s_{n,d} \xrightarrow{p} s^{\mathrm{true}}, \quad n \to \infty,\ d = d(n) \to \infty.\\
& && (s_{n,d}\text{: statistics from data}, \quad s^{\mathrm{true}}_{n,d}\text{: true value})
\end{aligned}$$

For example, Yata and Aoshima proved the presence of the curse of dimensionality in principal component analysis (PCA) and explicitly provided correction methods for eigenvalues of covariance matrix (corresponding to the variances of PCA) to mitigate its effects [2]:

$$\widetilde{\lambda}_i := \begin{cases} \lambda_i - \dfrac{1}{\min\{n-1,d\} - i + 1} \displaystyle\sum_{k=i+1}^{\min\{n-1,d\}} \lambda_k, & \text{for } i = 1, \dots, \ell, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where λ_i and $\widetilde{\lambda}_i$ are eigenvalues of the covariance matrix and their corrections, respectively. ℓ

is an upper bound in which the true data information is included. This collection theoretically improves the condition of convergence of the eigenvalues, indicating mitigation of the curse of dimensionality.

3 RECODE

To apply the eigenvalue modification theory into scRNA-seq data, we have formulated RECODE as follows:

$$\mathrm{RECODE}^{\mathrm{v1}}(X) := F^{-1}\left[U_{\ell^*}\widetilde{\Lambda}_{\ell^*}^{1/2}\Lambda_{\ell^*}^{-1/2}U_{\ell^*}^{\mathrm{T}}\left(F(X)-\overline{F(X)}\right)+\overline{F(X)}\right]$$

Here, $X \in \mathbb{Z}_{\geq 0}^{d\times n}$ is raw count matrix of scRNA-seq data. F is a noise variance-stabilizing normalization (NVSN), defined as:

$$F(x_{ij}) := \frac{x_{ij}}{t_j\sqrt{n^{-1}\sum_k x_{ik}t_k^{-2}}},$$

where $t_j = \sum_i x_{ij}$ is the total count for cell j. $U_{\ell^*} = (u_1,\ldots,u_{\ell^*}) \in \mathbb{R}^{d\times\ell^*}$ and $\Lambda_{\ell^*}^{-1/2} = \mathrm{diag}(\lambda_1^{-1/2},\ldots,\lambda_{\ell^*}^{-1/2})$ are matrices consisting of eigenvalues λ_i and eigenvectors u_i of the covariance matrix of $F(X) = (F(x_{ij}))$. $\widetilde{\Lambda}_{\ell^*}^{1/2} = \mathrm{diag}(\widetilde{\lambda}_1^{1/2},\ldots,\widetilde{\lambda}_{\ell^*}^{1/2})$ is a diagonal matrix of collections $\widetilde{\lambda}_i$ defined in Eq. (1).

One of the most significant contributions of this study is the introduction of NVSN F, which stabilizes the variance of noise generated in the data creation process. It bridged the gap between the theoretical conditions in Yata-Aoshima's method and the actual settings in the scRNA-seq data analysis, leading to the development of a theoretically grounded noise reduction method.

Acknowledgment

This research was partially supported by the JST CREST (Grant Number JPMJCR24Q1).

References

[1] Hall, P., Marron, J. S. and Neeman, A. Geometric representation of high dimension, low sample size data. J R Stat Soc B 67, 427-444 (2005).

[2] Yata, K. and Aoshima, M. Effective PCA for high-dimension, low-sample-size data with noise reduction via geometric representations. Journal of Multivariate Analysis 105, 193-215 (2012).

[3] Yata, K. and Aoshima, M. Automatic sparse PCA for high-dimensional data. Statistica Sinica 35, 1069-1090 (2025).

[4] Imoto, Y. et al. Resolution of the curse of dimensionality in single-cell RNA sequencing data analysis. Life Sci Alliance 5 (2022).

[5] Imoto, Y. Comprehensive Noise Reduction in Single-Cell Data with the RECODE Platform. bioRxiv (2024).

[6] Imoto, Y. Accurate highly variable gene selection using RECODE in scRNA-seq data analysis. bioRxiv (2025).

Long-time stability of time stepping schemes for the incompressible flow

Ming-Cheng Shiue[1,2]
[1]National Yang Ming Chiao Tung University, Taiwan, [2]Taiwan Society for Industrial and Applied Mathematics
e-mail : mcshiue@nycu.edu.tw

1 Introduction

To have more accurate and efficient numerical solution of incompressible flows governed by the Navier–Stokes equations remains a central challenge in computational fluid dynamics. Among various numerical concerns, long-time stability of time-stepping schemes is crucial, particularly in applications involving long-time integrations such as climate modeling and turbulent flow simulations. A time-stepping scheme is said to be L^2 long-time stable if the energy of the numerical solution remains uniformly bounded over arbitrarily long time intervals, in a manner that is consistent with the physical energy dissipation of the system. Meanwhile, a time-stepping scheme is said to be H^1 long-time stable if its enstrophy is uniformly bounded over arbitrarily long time intervals.

For the mathematical background, the Navier-Stokes equations on a bounded and smooth domain $\Omega \subseteq R^d (d = 2, 3)$ take the form

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \nu\Delta\boldsymbol{u} + \nabla p = f, \quad (x,t) \in \Omega \times (0,\infty), \\ \nabla \cdot \boldsymbol{u} = 0, \end{cases} \tag{1}$$

subject to the homogeneous Dirichlet boundary condition

$$\boldsymbol{u}\big|_{\partial\Omega} = 0, \tag{2}$$

and the initial data

$$\boldsymbol{u}(x,0) = \boldsymbol{u}_0(x), \quad x \in \Omega, \tag{3}$$

where the unknowns $\boldsymbol{u} = \boldsymbol{u}(x,t)$ and $p = p(x,t)$ represent the velocity and pressure of the fluid, respectively. The function f is assumed to be the external body force and $\nu > 0$ is the viscosity of the fluid. For the classical results, for $d = 2, 3$, we have

$$|\boldsymbol{u}|_{L^2} \le C_1, \quad \forall t > 0,$$

where a constant C_1 is dependent of the initial data $|\boldsymbol{u}_0|_{L^2}$ and $|f|_\infty$. For $d = 2$, we have

$$|\boldsymbol{u}|_{H^1} \le C_2, \quad \forall t > 0,$$

where a constant C_2 is dependent of the initial data $|\boldsymbol{u}_0|_{H^1}$ and $|f|_\infty$. We would like to know if these theoretical properties are preserved through the time discretization of the Navier-Stokes equations. In the literature, there have been a lot of works devoted to this study (see the references [1, 2, 3, 4, 5, 6, 7, 8]).

Proceedings of the 2025 Annual Meeting of the Japan Society for Industrial and Applied Mathematics

This presentation investigates a novel class of second-order time-stepping methods for the Navier-Stokes equations, specifically incorporating BDF and SAV-BDF approaches (see [8, 9]). The key findings reveal that these schemes maintain long-time L^2 stability without the restriction of the time step size. Moreover, when the initial data is sufficiently smooth, the algorithms achieve long-time H^1 stability without requiring the smallness condition on the time step size. These results confirm that the numerical methods accurately reproduce the asymptotic behavior of the Navier-Stokes dynamics, in full agreement with the established theoretical framework.

Acknowledgments ....
....

References

[1] S. Gottlieb, F. Tone, C. Wang, X.Wang, D. Wirosoetisno, Long-time stability of a classical efficient scheme for two-dimensional Navier-Stokes equations,SIAM Journal on Numerical Analysis, Volume 50 (2012), 126–150.

[2] T. Heister, M. A. Olshanskii, L. G. Rebholz, Unconditional long-time stability of a velocity-vorticity method for the 2D Navier-Stokes equations, Numerische Mathematik, Volume 135 (2017), 143–167.

[3] J. G. Heywood, R. Rannacher, Finite-element approximation of the nonstationary Navier-Stokes problem. Part (IV): error analysis for second-order time discretization, SIAM Journal on Numerical Analysis, Volume 27 (1990), 353–384.

[4] A. T. Hill, E. Süli, Approximation of the global attractor for the incompressible Navier-Stokes equations, IMA Journal of Numerical Analysis, Volume 20 (2000), 633-667.

[5] L. Rebholz, F. Tone, Long-time H^1-stability of BDF2 time stepping for 2D Navier–Stokes equations, Applied Mathematics Letters, Volume 141 (2023), 108624.

[6] F. Tone, D. Wirosoetisno, On the long-time stability of the implicit Euler scheme for the two-dimensional Navier-Stokes equations, SIAM Journal on Numerical Analysis, Volume 44 (2006), 29–40.

[7] D. Han, X. Wang, Long-time stable SAV-BDF2 numerical schemes for the forced Navier-Stokes equations, arXiv preprint arXiv:2410.06362, 2024.

[8] M.-C. Shiue, On the unconditional long-time L^2-stability of the BDF2 time stepping scheme for the two-dimensional Navier-Stokes equations, Applied Numerical Mathematics, Volume 214 (2025), 104–109.

[9] Q.-Y. Lin, M.-C. Shiue, On the unconditional long-time L2 stability of the SAV-BDF2 time-stepping schemes for the forced Navier–Stokes equations with nonsmooth initial data, Applied Mathematics Letters, Volume 171 (2025), 109665.